An Exact Algorithm for IP Traffic Engineering

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Mathematics for key technologies

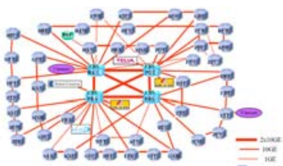
Overview

IP Network Optimization

Long-term:	Mid-term:	Short-term:
Network architecture	**Dimensioning**	**Traffic engineering**

Cooperation with DFN-Verein

- German national research and education network
- connects over 700 universities, research labs, ...
- over 5.000 TByte traffic per month (2007)

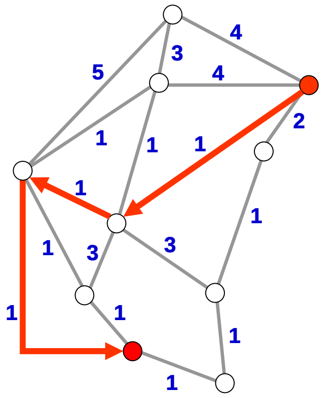


X-WiN IP topology, 02/2008

Our Algorithm:

- optimal (proven!!!) routing in backbone network
- incl. restoration paths, delay bounds, path restrictions, ...

Internet Routing



Intra-Domain: Shortest Path Routing

(OSPF, IS-IS, RIP, ...)

(1) Assign routing weights to links

(2) Send data along shortest paths

Variants

- Distance-vector vs. Link-state
- Single path vs. Multi-path

Routing control

- only **indirectly via lengths**
- only **all paths together**

Main challenges for planning

- all paths depend on same routing metric
- no description of valid path sets without routing weights

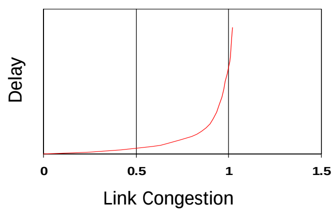
Traffic Engineering

Traffic Engineering:

Adjust routing to traffic demands

Goals: low packet loss rate
low delay
low jitter

Key figure: **link congestion**



TE planning problem (simplified)

Given: fixed network topology & capacities
traffic demands (forecast/measurement)

Task: **find/change routing weights** (only!!!)

Goal: **minimize maximum congestion**

Solution Methods I

Weight-based solution approaches

Modify weights ➡ Evaluate effects on routing

- Local search, genetic algorithms, simulated annealing, ...
  [Bley+98, FortzThorup00, FortzThorup04, FortzÜmit07, Farago+98, Ericsson+02, Buriol+05, ...]
- Lagrangian approaches
  [LinWang93, Bley03, ...]

... good solutions, but no or weak lower bounds.

Flow-based approaches

Optimize end-to-end paths and compatible routing weights

- Integrated MILP- or CP-Models
  [Bourquia+03, DeGiovanniFortzLabbe05, PioroTomaszewski+05, ParmarAhmedSokol05, ...]
  - Obtained by linearizing quadratic models
  - huge size, big-M coefficients, weak LP bounds

... almost hopeless for real-world problem sizes.

Solution Methods II

Weight-based solution approaches

Modify weights ➡ Evaluate effects on routing

- Local search, genetic algorithms, simulated annealing, ...
  [Bley+98, FortzThorup00, FortzThorup04, FortzÜmit07, Farago+98, Ericsson+02, Buriol+05, ...]
- Lagrangian approaches
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... good solutions, but no or weak lower bounds.

Flow-based approaches

Optimize end-to-end paths ⬌ Find compatible weights

- **Decomposition**
  [B.00, B.Koch 02, HolmbergYuan01, Prytz02, B. 2007, PioroTomaszewski2007, ...]
  - **Master:** optimize end-to-end paths (integer programming)
  - **Client:** find compatible routing weights (linear programming)

... proven optimal solutions for real-world problem sizes.

Decomposition Algorithm I

Integrated: Shortest path routing optimization

Variables

- routing weights
- path or arc-flows per demand
- link congestion

Constraints

- link capacity constraints
- flow conservation and integrality
- shortest path routing ← too many, too weak

Decomposition Algorithm II

Master: Routing path optimization

Variables

- ~~routing weights~~
- path or arc-flows per demand
- link congestion

Constraints

- link capacity constraints
- flow conservation and integrality
- shortest path routing (easy)
- ~~shortest path routing (hard)~~

Single path routing problem
(with some extra constraints)

Solved by branch-and-cut

- Specialized branching rules
- Problem specific heuristics
- Additional strong cuts

Collection of routing paths

hard shortest path routing constraint

Client: Find compatible weights
(or viol. shortest path routing constraint)

Variables

- routing weights

Constraints

- paths are unique shortest paths

Inverse shortest paths problem

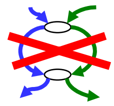
Solved by linear programming

- Feasible: compatible weights (scaling and rounding)
- Infeasible: violated constraint (LP-dual and Greedy)

Special Cuts

Clique inequalities for Bellman conflicts (subpath consistency)

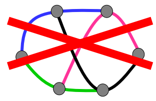
$$x_{P_1} + \ldots + x_{P_k} \leq 1 \quad \forall\, P_1, \ldots, P_k \text{ without B-property}$$



- most important 'easy' shortest path constraints

Rank inequalities for irreducible shortest path conflicts

$$\sum_{P \in S} x_P \leq |S| - 1 \quad \forall\, S \in \mathcal{C}_{USPS}$$

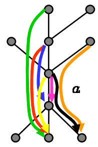


- 'hard' shortest path constraints
- **necessary and sufficient for correctness** of model
- separation for (near-) integer routings via client problem

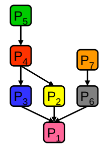
Induced knapsack cover inequalities

$$\sum_{P' \in C} x_{P'} \leq |C| - 1 \qquad \forall\, a,\ C \subseteq \{P : a \in P\} \text{ with } \sum_{P:\,P \text{ subpath of } P' \in C} d_{s_P, t_P} > u_a$$

- exploit subpath consistency and integrality
- **indispensible for good performance**



Paths



Precedences

Results I

- **proven optimal solutions** for small and medium-size networks
- very good solutions for large problems

	Name	$\|V\|$	$\|E\|$	$\|K\|$	LP	root	LB	UB	Nodes	Gap	Time
DFN	bwin	10	12	90	977	977	1000	1000	8	0.0	8
	gwin1	11	19	110	903	918	1000	1000	14	0.0	2
	gwin2	11	27	110	890	899	1000	1000	222	0.0	30
	gwin3	11	23	110	828	856	1000	1000	88	0.0	9
	gwin4	11	23	110	798	1000	1000	1000	2	0.0	1
	xwin1	42	58	250	958	958	1000	1000	7724	0.0	5679

real-world problems with symmetric single path routing and hop/delay restrictions, objective values scaled, path-flow formulation, times in seconds on P4 3.2 GHz

Results II

- **proven optimal solutions** for small and medium-size networks
- very good solutions for large problems

SNDlib

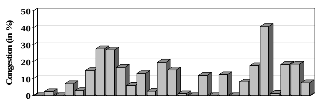
Problem	Nodes	Links	Demands	LP	LB	Sol	Nodes	Gap (%)	Time (s)
Atlanta	15	22	210	0.65	0.86	0.86	30	0.0	10.3
Dfn-bwin	10	45	90	0.34	0.69	0.69	89	0.0	26.5
Dfn-gwin	11	21	110	0.50	0.51	0.51	521	0.0	16.3
Di-yuan	11	42	22	0.25	0.62	0.62	33	0.0	1.8
France	25	45	300	0.60	0.71	0.74	76	5.0	10000.0
Germany50	50	88	662	0.64	0.64	0.73	56	12.7	10000.0
NewYork	16	49	240	0.44	0.62	0.62	15	0.0	54.9
Nobel-EU	28	41	378	0.44	0.44	0.45	75	0.3	10000.0
Nobel-GER	17	26	121	0.64	0.73	0.73	101	0.0	114.1
Nobel-US	14	21	91	0.48	0.49	0.49	77	0.0	20.4
Norway	27	51	702	0.54	0.54	0.62	99	14.9	10000.0
PDH	11	34	24	0.34	0.80	0.80	85	0.0	6.37
Polska	12	18	66	0.82	0.93	0.93	2149	0.0	200.2
TA1	24	55	396	0.30	0.93	0.93	11	0.0	289.2

benchmark instances from SNDlib with asymmetric demands, symmetric single path routing, no hop/delay restrictions; arc-flow formulation; times on P4 2.8 GHz

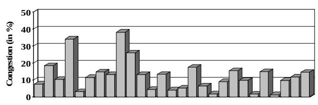
Practical Impact

Substantial load reduction by routing optimization!

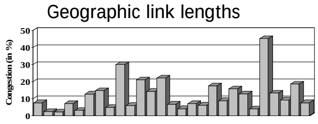
Default weights: $L_{max} > 38\,\%$



Inverse capacities



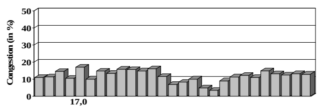
Geographic link lengths



Unit weights

Example: G-WiN 2 network

Optimized weights: $L_{max} = 17\,\%$



- Easy to implement new routing
- Immediate quality improvements
- More robust against traffic changes
- Less capacity expansion

Summary

Decomposition approach: MILP for routing paths + LP for weights

- Only practical method to compute proven optimal solutions
- Yields: optimal solutions for small & medium size problems
  best known solutions and bounds for large problems
- Applicable also for network topology and capacity planning

Software Implementation at ZIB / atesio GmbH:

- Variants and extensions:
  - Detailed node and link **hardware model**
  - Hop limits, delay limits, path constraints
  - Explicitly configured LSPs
  - Routing in **failure situations**
  - Several alternative MILP formulations
- Successfully used in practice for many years
  (German national research and education network)